Integration Formulas:-

Differentiation Formula	Indefinite Integral
$\frac{d}{dx}(k) = 0$	$\int k dx = \int k x^0 dx = kx + C$
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$
$\frac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x\, dx = -\csc x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$
$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
$\frac{d}{dx}\left(\sec^{-1}\lvert x\rvert\right) = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}\lvert x\rvert + C$

The Net Change Theorem:

It's an application of 2nd FTC, $\int_a^b F'(x)\,dx = \underbrace{F(b)-F(a)}_{\text{net change}}$

Recall: Velocity, $v(t) = s'(t)$, where $s(t)$ is displacement

Then we have, $\int_a^b v(t)\,dt = \underbrace{s(b)-s(a)}_{\substack{\text{net displacement} \\ \text{between time } a \text{ to } b.}}$

Example:- The velocity of a decelerating car is given by $v(t) = -\frac{1}{5}t + 20$. How far does the car travel between $t=0$ & $t=100$?

Solution:
$$\int_0^{100} v(t)\,dt = \int_0^{100}\left(-\frac{1}{5}t + 20\right)dt$$
$$= \left[-\frac{t^2}{10} + 20t\right]_0^{100}$$
$$= \left(-\frac{100^2}{10} + 20(100)\right) - \left(-\frac{0^2}{10} + 20(0)\right)$$
$$= 1000$$

More examples related to net change:

① Compute: $\int_0^8 e^{-x}\,dx$

Note:- $\frac{d}{dx}\left(-e^{-x}\right) = -\left(e^{-x}\cdot(-1)\right) = e^{-x}$

So, antiderivative of e^{-x} is $-e^{-x}$.

Hence, $\int_0^8 e^{-x}\,dx = \left[-e^{-x}\right]_0^8 = (-e^{-8}) - (-e^{-0})$

$= 1 - e^{-8}$

Note :- Total distance : $\int_a^b |v(t)|\,dt$.

(2) Suppose $v(t) = (t-1)^2(t-2)$ over $0 \leq t \leq 3$.

(a) What's the net displacement of the object between the interval $[0,3]$?

(b) What's the total displacement of the object between the interval $[0,3]$?

→ (a) Net displacement $= \int_0^3 v(t)\,dt$

$= \int_0^3 (t-1)^2(t-2)\,dt$

$= \int_0^3 (t^3 - 4t^2 + 5t - 2)\,dt$

$= \left[\frac{t^4}{4} - \frac{4t^3}{3} + \frac{5t^2}{2} - 2t\right]_0^3$

$= \left(\frac{3^4}{4} - \frac{4.3^3}{3} + \frac{5.3^2}{2} - 2.3\right) - (0)$

$= \frac{81}{4} - 42 + \frac{45}{2}$

$= \cancel{20} + \frac{1}{4} - \cancel{42} + \cancel{22} + \frac{1}{2}$

$= \frac{3}{4}$

$(t-1)^2(t-2)$

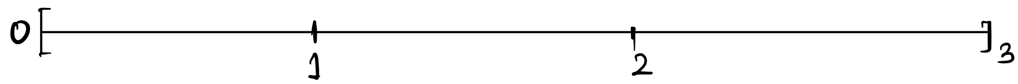
$= (t^2 - 2t + 1)(t-2)$

$= t^3 - 4t^2 + 5t - 2$

(b) We need $|v(t)|$, for that we need in which intervals $v(t)$ is +ve & where it is −ve.

Note:- $v(t) = 0 \Rightarrow (t-1)^2(t-2) = 0$
$\Rightarrow t = 1, 2$

Since, t represent time, it's $t \geq 0$.



For $0 \leq t < 1$, $v(t) < 0$
$1 < t < 2$, $v(t) < 0$] the object is moving backward.
$2 < t \leq 3$, $v(t) > 0$] the object is moving forward.

Total Distance = backward distance between $t=0$ & $t=2$ + forward distance between $t=2$ & $t=3$.

$$= \int_0^2 -v(t)\,dt + \int_2^3 v(t)\,dt$$

$$= \int_0^2 -(t-1)^2(t-2)\,dt + \int_2^3 (t-1)^2(t-2)\,dt$$

$$= \int_0^2 -(t^3-4t^2+5t-2)\,dt + \int_2^3 (t^3-4t^2+5t-2)\,dt$$

$$= -\left[\frac{t^4}{4} - \frac{4t^3}{3} + \frac{5t^2}{2} - 2t\right]_0^2 + \left[\frac{t^4}{4} - \frac{4t^3}{3} + \frac{5t^2}{2} - 2t\right]_2^3$$

$$= \frac{2}{3} + \frac{17}{12}$$

$$= \frac{25}{12}$$